ANISOTROPIC CREEP OF MATERIALS

O. V. Sosnin

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The materials used in equipment operating at high temperatures usually behave anisotropically with respect to the creep process. For certain materials, especially light alloys, the anisotropy resulting from previous processing cannot easily be removed by subsequent heat treatment and in designing parts it is necessary to take into account the real, and not the averaged properties of the material. Reference [1] presents experimental results on the creep of sheet material D1. In the interval of stresses and temperatures investigated it was shown that the material is orthotropic and its behavior is quite well described by the relation

$$\eta = B_i \sigma^n.$$

Here η is the creep strain, B_i and n are material constants, n being constant in all directions. This particular case of anisotropy is characterized by similarity of the creep strain rates for different directions and the same stresses, and, on account of the relative simplicity of the mathematical description, is most frequently mentioned in the periodical literature. This paper presents the results of experiments on AMG-3 material and provides some possibilities of describing the behavior under anisotropic creep conditions.

1. In describing the behavior of a material in steady-state creep under conditions of one-dimensional loading much use is made of empirical relations of the type

$$\eta = B\sigma^n, \qquad \eta = Ke^{\beta\sigma}. \tag{1.1}$$

The second equation is more convenient for describing anisotropic creep when in analytic expressions (1.1) both constants vary with the orientation of the test piece subjected to standard tensile or compressive testing. Henceforth we will assume that the behavior of the material under conditions of one-dimensional creep is well described by the second relation of type (1.1).

As noted previously [1], for visco-nonlinear process described in the one-dimensional case by the first relation of type (1.1), the experimental determination and comparison with theory of the transverse strain rate in tension and compression give a satisfactory confirmation of the assumed existence of a flow potential in the form of a function of a quadratic form of the stresses with coefficients depending on the properties of the material. It is natural to assume that a flow potential also exists in the more general case of anisotropy, when both constants vary in relations (1.1) describing the behavior of the material under one-dimensional loading. In this case we attempt to represent the flow potential analytically as a function of two quadratic forms of the stresses, i.e., we assume that the following relations hold:

$$\eta_{ij} = \frac{\partial \Phi}{\partial \sigma_{ij}}, \qquad \Phi = \Phi(T, S)$$
$$(T = a_{ijkl}\sigma_{ij}\sigma_{kl}, \quad S = b_{ijkl}\sigma_{ij}\sigma_{kl}). \tag{1.2}$$

Apart from any specific limitations, the potential function must satisfy two obvious conditions: a) the creep strain rate determined in terms of the potential from expression (1.2) must, in the case of one-dimensional loading, coincide with the second expression (1.1); b) in the case of vanishingly small anisotropy, when T and S are transformed into stress intensities, relation (1.2) must describe the creep process in accordance with the Mises criterion associated with its flow law.

Both these conditions will be satisfied if the potential is represented in the form

$$\Phi = \varphi\left(\frac{T}{S}\right) e^{\sqrt{T}}, \tag{1.3}$$

where φ is an arbitrary function of the ratio of the two quadratic forms. The strain rate tensor components will have the form

$$\eta_{ij} = e^{\sqrt{T}}\left(\varphi' \frac{T'S - S'T}{S^2} + \varphi \frac{T'}{2\sqrt{T}}\right)$$
$$\left(\varphi' = \frac{d\varphi}{d(T/S)}, \; T' = \frac{T\partial}{\partial \sigma_{ij}}, \; S' = \frac{\partial S}{\partial \sigma_{ij}}\right). \tag{1.4}$$

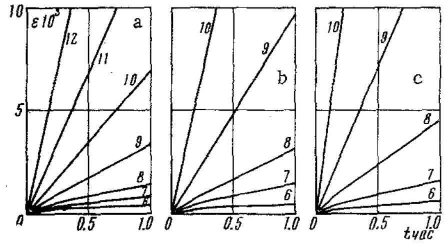

Fig. 1

The specific power dissipated in creep will be

$$W = \eta_{ij}\sigma_{ij} = \Phi\sqrt{T}. \tag{1.5}$$

In the case in question the surfaces Φ = const and W = const are not similar and the strain rate vectors, generally speaking, are not orthogonal to the surfaces of the constant specified dissipated power.

If the material behaves in relation to the creep process like an orthotropic incompressible medium, then, by analogy with [1], it is easy to establish that in the coordinate system associated with the principal axes of anisotropy quadratic forms T and S may be written, respectively, as

$$T = A_{11}(\sigma_{22} - \sigma_{33})^2 + A_{22}(\sigma_{33} - \sigma_{11})^2 + A_{33}(\sigma_{11} - \sigma_{22})^2 +$$
$$+ 2A_{12}\sigma_{12}^2 + 2A_{23}\sigma_{23}^2 + 2A_{31}\sigma_{31}^2,$$
$$S = B_{11}(\sigma_{22} - \sigma_{33})^2 + B_{22}(\sigma_{33} - \sigma_{11})^2 + B_{33}(\sigma_{11} - \sigma_{22})^2 +$$
$$+ 2B_{12}\sigma_{12}^2 + 2B_{23}\sigma_{23}^2 + 2B_{31}\sigma_{31}^2. \tag{1.6}$$

The coefficients of the quadratic forms A_{ij} and B_{ij} are determined from creep tests in simple tension or compression by comparing the second of expressions (1.1) and expression (1.4).

Thus, three series of experiments in the three principal directions of anisotropy, respectively, give

$$\beta_1 = \sqrt{A_{22} + A_{33}}, \qquad \beta_2 = \sqrt{A_{33} + A_{11}},$$
$$\beta_3 = \sqrt{A_{11} + A_{22}}, \qquad K_1 = \beta_1 \varphi\left(\frac{A_{22} + A_{33}}{B_{22} + B_{33}}\right),$$
$$K_2 = \beta_2 \varphi\left(\frac{A_{33} + A_{11}}{B_{33} + B_{11}}\right), \qquad K_3 = \beta_3 \varphi\left(\frac{A_{11} + A_{22}}{B_{11} + B_{22}}\right). \tag{1.7}$$

Hence, by assigning an analytic expression for the function φ, for example, setting $\varphi = MT/S$ or $\varphi = Me^{T/S}$, where M is a dimensionality constant, it is easy to determine the first three pairs of coefficients. The other three pairs can be determined from tensile or compressive creep tests on test pieces cut in directions not lying in the same plane and not coinciding with the principal directions of anisotropy of the test material.

Let, for example, a series of creep tests be conducted on test pieces cut in a direction forming an angle of 45° with the principal axis of anisotropy X_1 in one of the principal planes of anisotropy X_1X_2, and let the constants K' and β' in (1.1) be determined from this direction. The coefficients of the quadratic forms A_{ij} and B_{ij} are not invariant; their values depend on the choice of coordinate system. And since the quadratic forms T, S have their simplest form (1. 6) in the coordinate system associated with the principal axes of anisotropy, it is also convenient to refer the stress tensor components to the same coordinate system. Hence, making the necessary conversion and repeating the same arguments as in [1], we obtain

$$2\beta' = \sqrt{A_{11} + A_{22} + 2A_{12}}, \qquad K' = \beta'\varphi\left(\frac{A_{11} + A_{22} + 2A_{12}}{B_{11} + B_{22} + 2B_{12}}\right), \quad (1.8)$$

from which A_{12} and B_{12} are determined at known values of A_{11}, A_{22} and B_{11}, B_{22}.

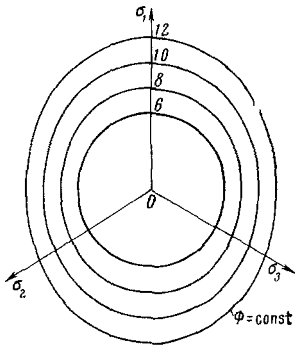

Fig. 2

Performing analogous operations in the other planes, we find the remaining coefficients of the quadratic forms and thus determine the flow potential (1. 2).

2. A typical material whose behavior under certain stress and temperature conditions is described in the case of one-dimensional loading by relations of type (1. 1), with both constants varying as a function of the direction in which the test piece is cut, is AMG-3 alloy taken in the initial state in the form of a rod 55 mm in diameter.

The test piece blanks were cut as follows; the rod was cut into right cylinders 50 and 15 mm tall and skew cylinders 15 mm tall with the axis inclined to the altitude at an angle of 45°. Four prismatic blanks measuring 15 × 15 mm^2 were cut from the center of the first cylinders for test pieces with axes coinciding with the direction of the axis of the initial rod; from the second cylinders we cut two blanks of the same size for test pieces with axes lying in arbitrary directions in the diametral plane of the starting rod; and from the third cylinders we cut two similar blanks for test pieces with axes inclined at an angle of 45° to the axis of the starting rod. From these blanks we made test pieces of the following type: a) for tensile tests—circular, with a gauge length of 20 mm and a diameter of 8 mm; b) for compressive tests—rectangular, with a cross section measuring 10 × 10 mm^2 and a gauge length of 35 mm. For each direction we prepared and tested ten pieces in tension and five in compression.

The tests were performed in the stress interval from 6 to 12 kg/ /mm^2 at 200° C. Some tests were performed at a stress of 5 kg/mm^2; these showed that at the given stress and below the creep process is only poorly described by relations (1. 1) owing to the clearly expressed hardening. With increase in the test stress level the first section of the creep curve is much reduced and at stresses of 8 kg/mm^2 and above is practically absent. In all the tests we measured the axial elongation and area reduction of the test pieces. Calculations of the three principal creep strains quite satisfactorily confirmed the hypothesis of incompressibility of the material under creep conditions, especially at high stress levels.

After stabilization of the temperature at 200° C, both tensile and the compressive tests were performed in accordance with the following four programs: 1) a load corresponding to the minimum stress σ = = 6 kg/mm^2 was applied and then, at intervals of 1 hr in the first three or four steps and at intervals of 0. 5 hr in the subsequent steps, the stress was increased by $\Delta\sigma$ = 1 kg/mm^2 to the maximum value of σ = 12 kg/mm^2; 2) the maximum load of σ = 12 kg/mm^2 was applied and then, at intervals of 0. 5 hr for the first two or three steps and 1 hr for the subsequent steps, the stress was reduced by $\Delta\sigma$ = 1 kg/mm^2 to the minimum value of σ = 6 kg/mm^2. The other two programs were similar, with the same time intervals but starting from a load of σ = 8 kg/mm^2, the stress then being increased or decreased by $\Delta\sigma$ = 1 kg/mm^2 to the maximum and minimum values, respectively.

In each time interval we recorded the creep curve, treating the application of the new stress as the initial state. The curves thus constructed for each stress in accordance with all four test programs, when superimposed, formed a quite narrow beam with the usual experimental scatter, but without any systematic deviation peculiar to any particular program. An exception was the last steps of the first and second programs, where the third stage of creep usually developed. These curves always passed above the center line of the beam and, as a rule, were not included in the calculations. This quite well confirms the hypothesis that the material behaves like a visco-nonlinear medium without hardening, and under the above-mentioned stress and temperature conditions and in a limited time interval its behavior does not depend on the loading history and consequently can be described by relations of type (1. 1) [2].

Figure 1 presents averaged values of the creep diagrams obtained for test pieces cut: a) along the axis of the rod, b) in the diametral plane of the rod, c) at an angle of 45° to the rod axis, where the numbers 6, 7, 8, etc. attached to the curves denote the stress at which the particular diagram was obtained. It is clear that at σ = 6 kg/mm^2 the creep curves for different directions almost coincide, i. e., at low stresses the material behaves like an isotropic material—the degree of anisotropy increases with increase in stress level. A similar effect was observed in light alloys by Johnson [3], but from a somewhat different standpoint.

As indicated above, the behavior of the material is quite well described by the second of relations (1. 1), and in analyzing the creep test data for test pieces cut in the above-mentioned three directions the following values of the constants were obtained:

$$\beta_1 = 0.753, \qquad \beta_2 = 1.022, \qquad \beta' = 1.097 \ [\mathrm{mm^2/kg}]$$
$$K_1 = 3.28 \cdot 10^{-6}, \qquad K_2 = 1 \cdot 10^{-6}, \qquad K' = 0.75 \cdot 10^{-6} \ [1/\mathrm{hr}^{-1}] \quad (2.1)$$

Here the subscript 1 denotes the direction along the axis, subscript 2 an arbitrary direction in the diametral plane of the rod, while a prime denotes the constants obtained from tests on test pieces cut at an angle of 45° to the axis from an arbitrary plane through the axis of the rod. Thus, under creep conditions the material behaves like a transversely isotropic medium with both constants in the second of expressions (1. 1) varying. The direction coinciding with the rod axis and any directions in the diametral plane of the rod will be principal directions of anisotropy. By virtue of the above-mentioned symmetry of the material properties, the number of independent coefficients in quadratic forms (1. 6) is reduced to three, which may be defined in terms of the experimental constants (2. 1). In fact, taking, for example,

$$\varphi\,(T/S) = M e^{T/S}, \qquad (2.2)$$

and substituting values of the constants from (2. 1) into (1. 7) and (1. 8), we find

$$M = 1 \ [\mathrm{kg/mm^2 \cdot hr}], \quad A_{11} = 0.761, \ A_{22} = A_{33} = 0.283,$$
$$A_{12} = A_{13} = 1.885 \ [\mathrm{mm^4/kg^2}], \ B_{11} = -5.251 \cdot 10^{-2}, \ B_{22} = B_{33} =$$
$$= -2.297 \cdot 10^{-2}, \ B_{12} = B_{13} = -13.179 \cdot 10^{-2} \ [\mathrm{mm^4/kg^2}]. \quad (2.3)$$

However, in the diametral plane, reasoning in the same way as in deriving relations (1. 7), we obtain for the coefficients A_{23} and B_{23} the expressions

$$2\beta_2 = \sqrt{A_{22} + A_{33} + 2A_{23}}, \qquad K_2 = \beta_2\varphi\left(\frac{A_{22} + A_{33} + 2A_{23}}{B_{22} + B_{33} + 2B_{23}}\right). \quad (2.4)$$

Hence, using (1.7), we find

$$A_{23} = 2A_{11} + A_{33} = 1.805 \ [\text{mm}^4/\text{kg}^2],$$
$$B_{23} = 2B_{11} + B_{33} = -12.799 \cdot 10^{-2} \ [\text{mm}^4/\text{kg}^2]. \qquad (2.5)$$

In the given case of anisotropy the ratios of the transverse strain rates no longer remain constant for the different stresses at which the one-dimensional creep tests are conducted. Thus, for example, in testing test pieces cut in the second direction, the ratio of transverse strain rates measured in the first and third directions in accordance with (1.4), using (2.2), will be

$$R \equiv \eta_{11}^{(2)} : \eta_{33}^{(2)} =$$
$$= \frac{2\sqrt{A_{11} + A_{33}}\,(A_{11}B_{33} - B_{11}A_{33}) - A_{33}(B_{11} + B_{33})^2\sigma}{2\sqrt{A_{11} + A_{33}}\,(B_{11}A_{33} - A_{11}B_{33}) - A_{11}(B_{11} + B_{33})^2\sigma}, \qquad (2.6)$$

i.e., a linear-fractional function of the stresses.

Below we present values of R^* calculated from (2.6) and R° measured experimentally. The satisfactory agreement between the experimental and calculated data on the transverse strain ratio for different stresses gives a certain indirect confirmation of the hypothesis of the existence of a flow potential (1.2) for anisotropic media and its use in technical calculations.

σ, kg/mm^2 = 6	7	8	9	10
R^* = 0.80	0.72	0.67	0.63	0.60
R° = —	0.65	0.63	0.57	0.50

Figure 2 shows projections of the potential flow surfaces Φ = const in the deviatoric plane, calculated for the given material at stress levels corresponding to the process of creep along the first principal direction of anisotropy at σ = 6, 8, 10, and 12 kg/mm^2.

As might be expected, at σ = 6 kg/mm^2 the surface Φ = const is close to a circular cylinder, and its projection in the deviatoric plane almost coincides with the Mises circle. The anisotropy becomes manifest as the stress level increases, as reflected by the change in the shape of the contour Φ = const in the deviatoric plane. Although the diametral plane X_2X_3 of the rod is also a plane of isotropy with respect to the creep properties under one-dimensional loading, the projections on the deviatoric plane of part of the surfaces Φ = const in the sectors $-\sigma_3$, σ_2 and $-\sigma_2$, σ_3 are not segments of arcs of circles: all the contours in the deviatoric plane are compressed in a direction perpendicular to the projection of σ_1 axis on that plane.

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